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# A note on the growth of Kelvin–Helmholtz waves on thin liquid sheets

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A photographic study has been made of Kelvin–Helmholtz waves on thin liquid sheets and it has been found that, contrary to two-dimensional theory, wave growth is critically dependent upon sheet velocity and distance from the origin. This is attributed to boundary-layer separation and to subsequent vortex growth.

Aerodynamic, or Kelvin–Helmholtz, wave growth on liquid sheets has been studied by a number of workers (Squire 1953; Hagerty & Shea 1955; Clark & Dombrowski 1972) and the resulting analyses have helped, in broad outline, to explain a number of observed phenomena. All theories agree that, from an initial small amplitude, wave growth follows an exponential law, but no attempt has so far been made to examine the validity of this prediction. Published photographs do not provide adequate material for this purpose since, apart from some limited experiments by Hagerty & Shea, observations have been made only with natural waves of random frequency and size.

In a recent study, to be described in detail later, sinuous waves of controlled frequency and amplitude have been imposed on thin sheets of water to determine optimum conditions for maximum growth rate, and to compare the results with calculations based on theory. A number of single-orifice fan spray nozzles similar to those studied previously (Clark & Dombrowski 1972) were employed, and perturbations were induced on the resulting fan-shaped sheets by vibrating each nozzle normal to the plane of the sheet. The waves were subsequently analysed from photographs taken along the plane of the sheet.

The results were unexpected. The existing two-dimensional theory predicts waves which grow exponentially with time, that is to say, with distance from the origin in the experimental set-up. We have discovered a situation in which the wave growth depends critically on the sheet velocity and distance from the nozzle. At a relatively high velocity (see figure 1, plate 1) wave growth proceeds until breakdown occurs.† At lower velocities however (figure 2, plate 2), the rate of growth is at first large but then rapidly diminishes until a maximum amplitude is reached, after which the amplitude may actually diminish. Figure 2 demonstrates that the phenomenon appears to be associated with the waves reaching very large

† Inspection of the views illustrated in figure 1 shows that the stream of drops formed along the axis of the sheet in figure 1 (*b*) results from the breakdown of the rims of the sheet.

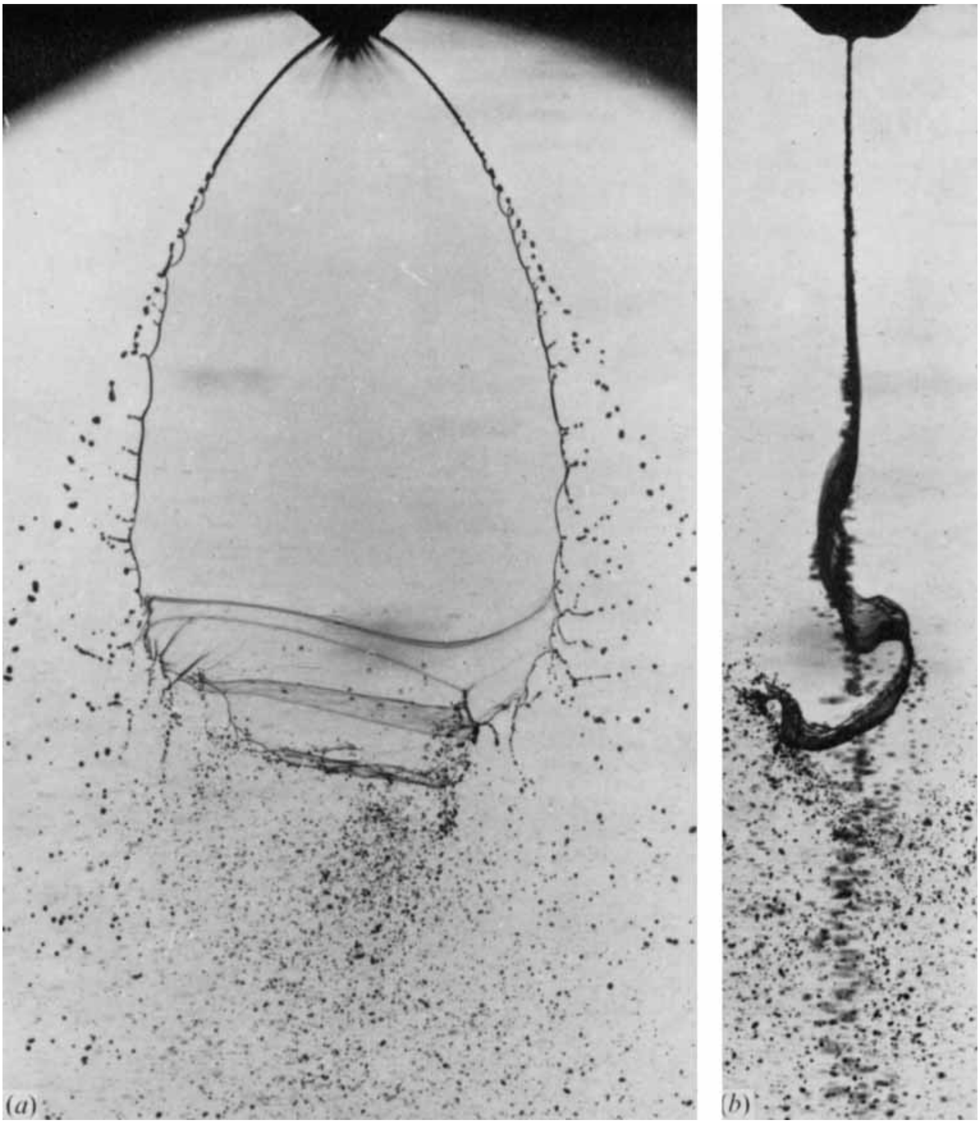


FIGURE 1. Wave growth on high velocity liquid sheet. (2·5 × magnification.)

FIGURE 2. Wave growth on low velocity liquid sheet. (2·5 × magnification.)

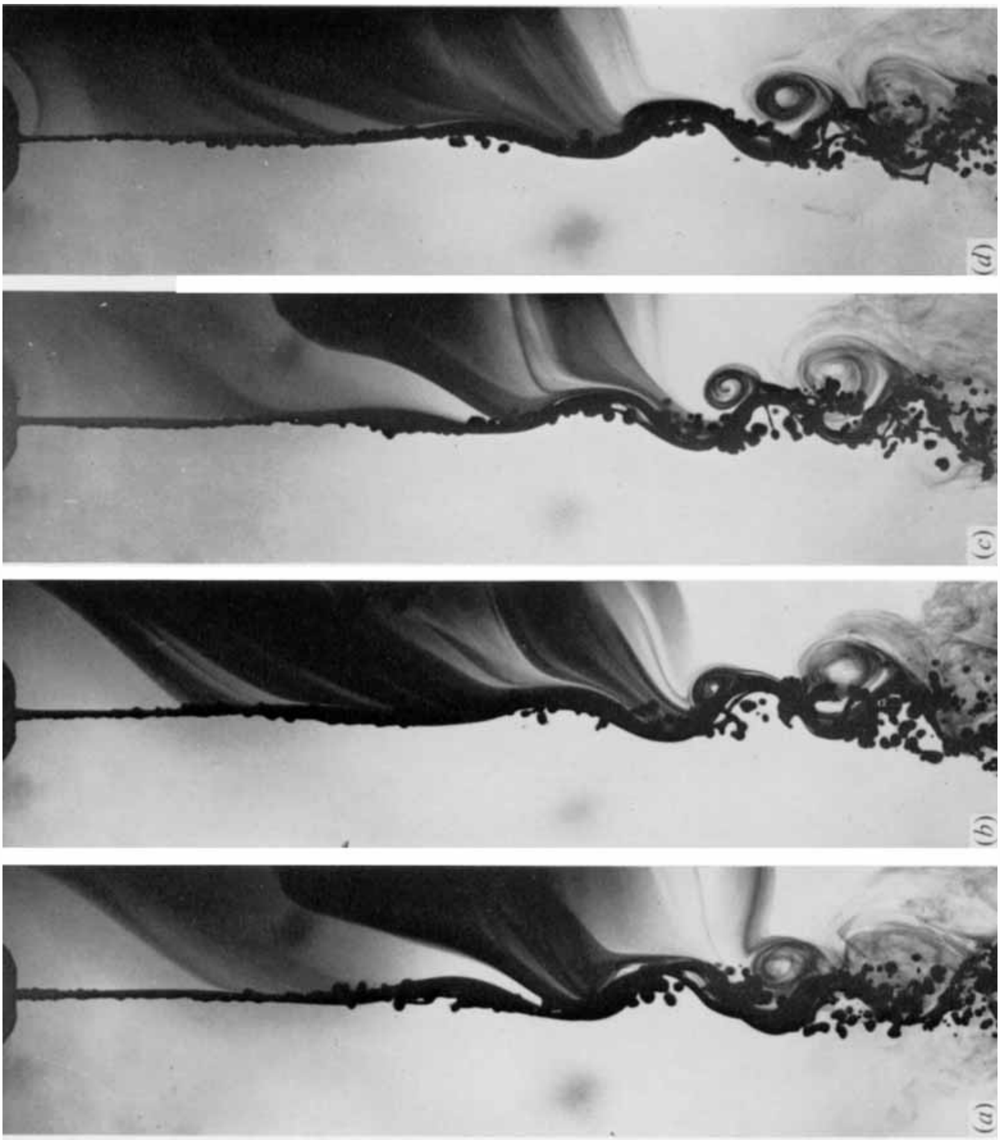


FIGURE 3. Vortex growth on low velocity liquid sheet. (2·5 × magnification.)

FIGURE 4. Vortex growth on high velocity liquid sheet. (2·5 × magnification.)

amplitude-to-wavelength ratios, and becoming highly non-sinusoidal in form. It was concluded that the cause must lie in the nature of the air flow pattern around the sheet. Consequently a study was made of the streamlines adjacent to the sheet, using titanium tetrachloride smoke tracers and a flash stroboscope synchronized in variable phase with the vibrator.

It was found that, for a given operating condition, a vortex was formed upstream of each wave crest at a constant distance from the nozzle, and it rapidly increased in size. Photographs were then taken with this system although, since the available camera shutter had a maximum speed of only 0·01 s, single flashes per picture could be achieved only at wave frequencies below 200 Hz; under most conditions each photograph was made up of a number of superimposed pictures. A series of four photographs, demonstrating the growth of a vortex on one side of the sheet, is given in figure 3 (plate 3). These show that the boundary layer breaks away from the surface of the wave when appreciable growth has occurred and clearly indicate the subsequent growth and movement of the vortex. The phenomenon follows a regular cyclic path and this is exemplified in the photographs by the fact that each corresponds to four superimposed pictures. The extent to which the vortex grows depends upon the operating conditions. At high growth rates where the sheet breaks down rapidly, vortex growth is minimal and extends over only a small portion of the wave. An example of this is shown in figure 4 (plate 4). It will be noticed that the formation of the vortices is similar to those cast off by a solid plate moved in a direction perpendicular to its plane and, since vortices are produced on both sides of the sheet, the overall pattern resembles a Kármán vortex street.

It is hoped to present a preliminary analysis of this phenomenon shortly.

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